Beamforming Amplify-and-Forward Relay Networks With Feedback Delay and Interference

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***Abstract*—In this letter, we investigate the effect of feedback delay on the performance of a dual-hop amplify-and-forward (AF) relay network with beamforming in the presence of multiple interferers over Rayleigh fading channels. Specifically, we derive closed-form expressions for the outage probability (OP) and the symbol error rate (SER). Furthermore, to render insights into the effect of feedback delay and interference on the network performance, asymptotic OP and SER are also presented. These asymptotic expressions are very tight in the high signal-to-noise ratio regime, readily enabling us to obtain the diversity and coding gains of the considered network.**

***Index Terms*—Amplify-and-forward (AF), beamforming, feedback delay, interference, outage probability, symbol error rate.**

I. Introduction

BEAMFORMING transmission for amplify-and-forward (AF) relay networks, which has been known as a means of obtaining full diversity gain, has been widely studied in the literature (see, e.g., [1]–[6], and the references therein). Specifically, the performance of dual-hop relay networks using transmit beamforming at the source with maximum ratio combining (MRC) at the destination has been presented in [1], [2]. In [6], the impact of feedback delay on the performance of dual-hop AF relay networks with beamforming over Rayleigh fading channels has been investigated.

In a variety of practical scenarios in relay networks, along with the presence of feedback delay, interference considerably diminishes the network performance [7]–[9]. Although feedback delay in AF relay networks with beamforming transmission has been independently investigated in [6], the effect of interference on the network performance remains unanswered. Furthermore, the joint effect of feedback delay and interference has not been established yet.

In this letter, we investigate the effect of feedback delay and interference on dual-hop AF relay networks with beamforming transmission. In particular, we derive closed-form expressions for the outage probability (OP) and the symbol error rate (SER) of the considered network. Asymptotic expressions for the OP and SER are also obtained to provide insights into the diversity behavior. Based on our asymptotic expressions, fundamental insights are reached. First, under the effect of interference (with no feedback delay), the diversity gain is equal to the minimum of the number of antennas at the source and the destination. Second, under the combined effect of interference and feedback delay, the diversity gain is equal to unity.

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II. System and Channel Model

Consider a dual-hop AF relay network with L_1 antennas at source S, a single antenna at relay R, and L_2 antennas at destination D. We assume that R operates in an interference environment. This assumption applies to cellular networks, where R operates at the cell edge to extend service coverage. Therefore, the received signal at R is impaired by co-channel interference. The communication between S and D occurs over two hops. In the first hop, S transmits symbol $s_0(t)$ to R with power $P_0 = \mathbb{E}\{|s_0(t)|^2\}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. In the second hop, R amplifies and retransmits its received signal to D with power P_r.

In the first hop, the received signal at R is compromised by N_I interferers and additive white Gaussian noise (AWGN) as

$$y_{\mathrm{R}}(t) = \mathbf{h}_1^H(t)\mathbf{w}_1(t)s_0(t) + \sum_{l=1}^{N_I} h_{3l}(t)s_l(t) + n_1(t) \quad (1)$$

where $\mathbf{h}_1(t) = [h_{11}(t), \ldots, h_{1L_1}(t)]^T$ is the Rayleigh channel coefficient vector from S to R, with $\mathbb{E}\{|h_{1j}(t)|^2\} = \Omega_1$ for $j = 1, \ldots, L_1$ and $n_1(t)$ is the AWGN with zero-mean and variance σ_1^2. In (1), $\{h_{3l}(t)\}_{l=1}^{N_I}$ are the Rayleigh channel coefficients of N_I interferers at R with $\mathbb{E}\{|h_{3l}(t)|^2\} = \Omega_{3l}$, and $\{s_l(t)\}_{l=1}^{N_I}$ are the interfering signals with $P_l = \mathbb{E}\{|s_l(t)|^2\}$. The beamforming vector at S is defined as $\mathbf{w}_1(t) = \mathbf{h}_1(t-\tau)/\|\mathbf{h}_1(t-\tau)\|_F$, where τ is the feedback delay and $\|\cdot\|_F$ is the Frobenius norm.

In the second hop, R forwards $y_{\mathrm{R}}(t)$ to D after applying a scale factor β. Assuming CSI-assisted AF relaying, we have

$$\beta^2 = P_r\left[P_0|\mathbf{h}_1^H(t)\mathbf{w}_1(t)|^2 + \sum_{l=1}^{N_I} P_l|h_{3l}(t)|^2\right]^{-1}. \quad (2)$$

The received signal at D after multiplied with beamforming vector $\mathbf{w}_2^H(t)$ can be written as

$$\begin{aligned} y_{\mathrm{D}}(t) &= \beta\mathbf{w}_2^H(t)\mathbf{h}_2(t)\mathbf{h}_1^H(t)\mathbf{w}_1(t)s_0(t) \\ &+\beta\mathbf{w}_2^H(t)\mathbf{h}_2(t)\left(\sum_{l=1}^{N_I} h_{3l}(t)s_l(t) + n_1(t)\right) + \mathbf{w}_2^H(t)\mathbf{n}_2(t) \end{aligned} \quad (3)$$

where $\mathbf{h}_2(t) = [h_{21}(t), \ldots, h_{2L_2}(t)]^T$ is the Rayleigh channel coefficient vector from R to D, with channel mean power $\mathbb{E}\{|h_{2j}(t)|^2\} = \Omega_2$ for $j = 1, \ldots, L_2$, $\mathbf{w}_2(t) = \mathbf{h}_2(t)/\|\mathbf{h}_2(t)\|_F$, and $\mathbf{n}_2(t)$ is an $L_2 \times 1$ AWGN vector whose elements are complex Gaussian random variables (RVs) with zero-mean and variance σ_2^2. Assuming that the noise power at the relay is much smaller than the interference power such that its effect can be neglected, the end-to-end

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signal-to-interference plus noise ratio (SINR) can be obtained as [7], [8]

$$\gamma_{\rm D} = \frac{\gamma_1\gamma_2}{\gamma_1 + (\gamma_2+1)\gamma_3} \tag{4}$$

where $\gamma_1 = P_0|\mathbf{h}_1^H(t)\mathbf{w}_1(t)|^2$, $\gamma_2 = P_r\|\mathbf{h}_2(t)\|_F^2/\sigma_2^2$, and $\gamma_3 = \sum_{l=1}^{N_I} P_l|h_{3l}(t)|^2$.

III. End-to-End Performance Analysis

A. Outage Probability

OP is defined as the probability that the end-to-end SINR $\gamma_{\rm D}$ falls below a predefined threshold $\gamma_{\rm th}$. Hence, our aim is to derive the cumulative distribution function (CDF) of $\gamma_{\rm D}$, $F_{\gamma_{\rm D}}(\gamma)$. Given (4), $F_{\gamma_{\rm D}}(\gamma)$ can be written as

$$F_{\gamma_{\rm D}}(\gamma) = F_{\gamma_2}(\gamma) + \int_0^\infty\int_\gamma^\infty F_{\gamma_1}(\phi)f_{\gamma_2}(\gamma_2)f_{\gamma_3}(\gamma_3)d\gamma_2 d\gamma_3 \tag{5}$$

where $\phi = \gamma(\gamma_2+1)\gamma_3/(\gamma_2-\gamma)$. To evaluate $F_{\gamma_{\rm D}}(\gamma)$, we require the CDF of γ_1 and the probability density function (PDF) of γ_2 and γ_3. Under the time-varying channel, the relationship between $\mathbf{h}_1(t)$ and $\mathbf{h}_1(t-\tau)$ can be expressed as [10, eq. (6)], $\mathbf{h}_1(t) = \rho\mathbf{h}_1(t-\tau) + \sqrt{1-|\rho|^2}\mathbf{e}_1(t)$, where ρ is the normalized correlation coefficient between $h_{1j}(t)$ and $h_{1j}(t-\tau)$, for $j = 1,\ldots,L_1$. Here, $\mathbf{e}_1(t)$ denotes an $L_1\times 1$ error vector, whose elements are complex Gaussian RVs with zero mean and variance Ω_1. For Jake's fading spectrum, we have $\rho = J_0(2\pi f_d\tau)$ where f_d is the Doppler frequency and $J_n(\cdot)$ is the Bessel function of the first kind [11, eq. (8.441.1)]. Using [11, eq. (3.351.1)] and [10, eq. (15)], we obtain

$$F_{\gamma_1}(\gamma_1) = 1 - \sum_{k=0}^{L_1-1}\sum_{p=0}^{L_1-k-1}\frac{\beta_{kp}}{\bar\gamma_1^{L_1+p}}e^{-\gamma_1/\bar\gamma_1}\gamma_1^p = 1-\psi(\gamma_1) \tag{6}$$

where $\beta_{kp} = C_k^{L_1-1}(|\rho|^2)^{L_1-k-1}\left[\bar\gamma_1(1-|\rho|^2)\right]^k\bar\gamma_1^{L_1-k}/p!$, $C_k^n = \binom{n}{k}$, and $\bar\gamma_1 = P_0\Omega_1$. In addition, the PDFs of γ_2 and γ_3 are given by

$$f_{\gamma_2}(\gamma_2) = \frac{\gamma_2^{L_2-1}}{\bar\gamma_2^{L_2}\Gamma(L_2)}e^{-\gamma_2/\bar\gamma_2} \tag{7}$$

$$f_{\gamma_3}(\gamma_3) = \sum_{l=1}^{N_I}\frac{\eta_l}{\bar\gamma_{3l}}e^{-\gamma_3/\bar\gamma_{3l}} \tag{8}$$

respectively, where $\bar\gamma_2 = P_r\Omega_2/\sigma_2^2$, $\bar\gamma_{3l} = P_l\Omega_{3l}$, $\Gamma(n)$ is the gamma function [11, eq. (8.310.1)], and

$$\eta_l = \left[\prod_{u=1,u\neq l}^{N_I}\frac{\bar\gamma_{3u}^{-1}}{(s+\bar\gamma_{3u}^{-1})}\right]_{s=-\bar\gamma_{3l}^{-1}}. \tag{9}$$

Substituting (6), (7), and (8) into (5), $F_{\gamma_{\rm D}}(\gamma)$ is expressed as

$$F_{\gamma_{\rm D}}(\gamma) = 1 - \int_0^\infty\int_0^\infty \psi(\theta)f_{\gamma_2}(\gamma_2+\gamma)d\gamma_2 f_{\gamma_3}(\gamma_3)d\gamma_3 \tag{10}$$

where $\theta = \gamma(\gamma_2+\gamma+1)\gamma_3/\gamma_2$. After some algebraic manipulations and with the help of [11, eq. (3.471.9)], the inner integral in (10) is expressed as

$$\int_0^\infty \psi(\theta)f_{\gamma_2}(\gamma_2+\gamma)d\gamma_2$$

$$= \frac{2}{\bar\gamma_1^{L_1}}\sum_{k=0}^{L_1-1}\sum_{p=0}^{L_1-k-1}\frac{\beta_{kp}\gamma^p\gamma_3^p}{\bar\gamma_1^p}\times e^{-\gamma\gamma_3/\bar\gamma_1}\sum_{q=0}^{p}\frac{C_q^p(\gamma+1)^q}{\bar\gamma_2^{L_2}\Gamma(L_2)}$$
$$\cdot\sum_{r=0}^{L_2-1}C_r^{L_2-1}\gamma^{L_2-1-r}e^{-\gamma/\bar\gamma_2}\gamma_3^{(r-q+1)/2}$$
$$\times\left(\frac{\bar\gamma_2\gamma^2+\bar\gamma_2\gamma}{\bar\gamma_1}\right)^{(r-q+1)/2}\mathcal{K}_{r-q+1}\left(2\sqrt{\frac{\gamma^2+\gamma}{\bar\gamma_1\bar\gamma_2}}\gamma_3\right) \tag{11}$$

where $\mathcal{K}_n(\cdot)$ is the nth order modified Bessel function of the second kind [11, eq. (8.432.1)]. By substituting (8) and (11) into (10) and using [11, eq. (6.643.3)], we obtain

$$F_{\gamma_{\rm D}}(\gamma)$$
$$= 1 - \frac{1}{\bar\gamma_1^{L_1-1}}\sum_{k=0}^{L_1-1}\sum_{p=0}^{L_1-k-1}\beta_{kp}\sum_{q=0}^{p}\frac{C_q^p}{\Gamma(L_2)}$$
$$\times\sum_{r=0}^{L_2-1}C_r^{L_2-1}e^{-\gamma/\bar\gamma_2}\sum_{l=1}^{N_I}\frac{\eta_l}{\bar\gamma_{3l}}\frac{\Gamma(p+1)\Gamma(p+r-q+2)}{\bar\gamma_2^{(2L_2-r+q-2)/2}}$$
$$\times\gamma^{(2L_2+2p-r-q-2)/2}(1+\gamma)^{(r+q)/2}$$
$$\cdot\left(\frac{\bar\gamma_{3l}\gamma+\bar\gamma_1}{\bar\gamma_{3l}}\right)^{-(2p+r-q+2)/2}$$
$$\times e^{(\bar\gamma_{3l}(\gamma^2+\gamma))/2\bar\gamma_2(\bar\gamma_{3l}\gamma+\bar\gamma_1)}\mathcal{W}_{-(2p+r-q+2)/2,(r-q+1)/2}$$
$$\cdot\left(\frac{\bar\gamma_{3l}(\gamma^2+\gamma)}{\bar\gamma_2(\bar\gamma_{3l}\gamma+\bar\gamma_1)}\right) \tag{12}$$

where $\mathcal{W}_{a,b}(.)$ is the Whittaker function [11, eq. (9.222)]. The OP is readily obtained by substituting $\gamma = \gamma_{\rm th}$ into (12).

B. Symbol Error Rate

In the sequel, we derive a closed-form analytical expression for the SER. From (4), we express the upper bound on the SINR as $\gamma_{\rm DUB} = \min\{\gamma_{13},\gamma_2\}$, where $\gamma_{13} = \gamma_1/\gamma_3$. As a result, the SER in the high signal-to-noise ratio (SNR) regime can be approximated as [1]

$$P_E \approx \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty F_{\gamma_{\rm DUB}}(\gamma)\gamma^{-1/2}e^{-b\gamma}d\gamma \tag{13}$$

where $a,\ b$ are modulation parameters [1]. To evaluate the SER, we first derive the CDF of $\gamma_{\rm DUB}$ as $F_{\gamma_{\rm DUB}}(\gamma) = 1-(1-F_{\gamma_{13}}(\gamma))(1-F_{\gamma_2}(\gamma))$. Additionally, $F_{\gamma_{13}}(\gamma)$ is given by

$$F_{\gamma_{13}}(\gamma)$$
$$= \int_0^\infty F_{\gamma_1}(\gamma\gamma_3)f_{\gamma_3}(\gamma_3)d\gamma_3$$
$$= 1 - \sum_{k=0}^{L_1-1}\sum_{p=0}^{L_1-k-1}\frac{\beta_{kp}\Gamma(p+1)}{\bar\gamma_1^{L_1+p}}\gamma^p\sum_{l=1}^{N_I}\frac{\eta_l}{\bar\gamma_{3l}}\left(\frac{\bar\gamma_1\bar\gamma_{3l}}{\bar\gamma_{3l}\gamma+\bar\gamma_1}\right)^{p+1}. \tag{14}$$

By utilizing $F_{\gamma_{13}}(\gamma)$ in (14) together with $F_{\gamma_2}(\gamma)$ which can be obtained from (7), $F_{\gamma_{\rm DUB}}(\gamma)$ can be expressed as

$$F_{\gamma_{\rm DUB}}(\gamma) = 1 - \sum_{k=0}^{L_1-1}\sum_{p=0}^{L_1-k-1}\frac{\beta_{kp}\Gamma(p+1)}{\bar\gamma_1^{L_1-1}}\sum_{l=1}^{N_I}\eta_l\bar\gamma_{3l}^p$$
$$\times\sum_{q=0}^{L_2-1}\frac{\bar\gamma_2^{-q}}{q!}\frac{\gamma^{p+q}}{(\bar\gamma_{3l}\gamma+\bar\gamma_1)^{p+1}}e^{-\bar\gamma_2^{-1}\gamma}. \tag{15}$$

Therefore, by substituting (15) into (13) and with the help of [12, eq. (2.3.6.9)], a closed-form expression for the SER is expressed as

$$P_E \approx \frac{a}{2} - \frac{a\sqrt{b}}{2\sqrt{\pi}} \sum_{k=0}^{L_1-1} \sum_{p=0}^{L_1-k-1} \frac{\beta_{kp}\Gamma(p+1)}{\bar{\gamma}_1^{L_1-1}} \sum_{l=1}^{N_I} \eta_l \sum_{q=0}^{L_2-1} \frac{\bar{\gamma}_2^{-q}}{q!} \times \bar{\gamma}_1^{q-1/2} \bar{\gamma}_{3l}^{-q-1/2} \Gamma\left(p+q+\frac{1}{2}\right) \times U\left(p+q+\frac{1}{2}, q+\frac{1}{2}; \frac{\bar{\gamma}_1(b+\bar{\gamma}_2^{-1})}{\bar{\gamma}_{3l}}\right) \tag{16}$$

where $U(a,b;x)$ is the confluent hypergeometric function [11, eq. (9.211.4)].

IV. High SNR Performance Analysis

A. Outage Probability at High SNR

The closed-form expression in (12) provides the exact OP evaluation for any given SNR rather than the network diversity behavior. To further render insights into the diversity performance, the asymptotic analysis for the OP is presented. Assuming that $\bar{\gamma}_1$ and $\bar{\gamma}_2$ approach to infinity with constant ratio $\mu = \bar{\gamma}_2/\bar{\gamma}_1$, the asymptotic OP can be deduced from Theorem 1.

Theorem 1: Considering only the interference (no feedback delay), i.e., $\rho = 1$, the asymptotic OP is given by

$$P_{\text{out}}^{\infty} \approx \begin{cases} \sum_{l=1}^{N_I} \eta_l \bar{\gamma}_{3l}^{L_1} \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{L_1}, & L_1 < L_2 \\ \sum_{l=1}^{N_I} \frac{\eta_l}{L_2!\mu^{L_2}} \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{L_2}, & L_1 > L_2 \\ \sum_{l=1}^{N_I} \eta_l \left(\bar{\gamma}_{3l}^{L} + \frac{1}{L!\mu^L}\right) \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{L}, & L_1 = L_2 = L \end{cases}. \tag{17}$$

Considering both interference and feedback delay, i.e., $\rho < 1$, the asymptotic OP is written as

$$P_{\text{out}}^{\infty} \approx (1-|\rho|^2)^{L_1-1} \sum_{l=1}^{N_I} \eta_l \bar{\gamma}_{3l} \frac{\gamma_{\text{th}}}{\bar{\gamma}_1}. \tag{18}$$

Proof: First, from [6], it is noted that the McLaurin series of the CDF of $\gamma_{\text{DUB}} = \min\{\gamma_{13}, \gamma_2\}$ has the same first nonzero coefficient as that of the CDF of γ_{D}. We now consider the following two cases of interest. Case 1: interference only (no feedback delay), i.e., $\rho = 1$, and Case 2: interference and feedback delay, i.e., $\rho < 1$. For Case 1, $F_{\gamma_{\text{DUB}}}(\gamma)$ is given by

$$F_{\gamma_{\text{DUB}}}(\gamma) = 1 - \sum_{p=0}^{L_1-1} \left(\frac{\gamma}{\bar{\gamma}_1}\right)^p \sum_{l=1}^{N_I} \frac{\eta_l}{\bar{\gamma}_{3l}} \left(\frac{\gamma}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_{3l}}\right)^{-p-1} \times e^{-\gamma/\bar{\gamma}_2} \sum_{q=0}^{L_2-1} \frac{1}{q!} \left(\frac{\gamma}{\bar{\gamma}_2}\right)^q. \tag{19}$$

Denoting $z = \gamma/\bar{\gamma}_1$, $\bar{\gamma}_2 = \mu\bar{\gamma}_1$, after some calculations, $F_{\gamma_{\text{DUB}}}(\gamma)$ is rewritten as

$$F_{\gamma_{\text{DUB}}}(z) = 1 - \sum_{l=1}^{N_I} \frac{\eta_l}{\bar{\gamma}_{3l} z} \sum_{p=0}^{L_1-1} \left(\frac{\bar{\gamma}_{3l} z}{\bar{\gamma}_{3l} z + 1}\right)^{p+1} \times e^{-z/\mu} \sum_{q=0}^{L_2-1} \frac{1}{q!} \left(\frac{z}{\mu}\right)^q. \tag{20}$$

Using some algebraic manipulations and the McLaurin series expansion for the exponential function in (20), we have

$$F_{\gamma_{\text{DUB}}}(z) \overset{z\to 0}{\approx} 1 - \sum_{l=1}^{N_I} \eta_l \left(1 - \bar{\gamma}_{3l}^{L_1} z^{L_1}\right) \left(1 - \frac{z^{L_2}}{L_2!\mu^{L_2}}\right). \tag{21}$$

For Case 2, we rewrite the CDF $F_{\gamma_{\text{DUB}}}(\gamma)$ as

$$F_{\gamma_{\text{DUB}}}(z) = 1 - \sum_{k=0}^{L_1-1} \sum_{p=0}^{L_1-k-1} C_k^{L_1-1} |\rho|^{2(L_1-k-1)} (1-|\rho|^2)^k \times \sum_{l=1}^{N_I} \eta_l \frac{(\bar{\gamma}_{3l} z)^p}{(\bar{\gamma}_{3l} z + 1)^{p+1}} \sum_{q=0}^{L_2-1} \frac{1}{q!} \left(\frac{z}{\mu}\right)^q e^{-z/\mu}. \tag{22}$$

By performing the McLaurin series expansion for the exponential function in (22), after some algebraic manipulations, we get

$$F_{\gamma_{\text{DUB}}}(z) = 1 - \sum_{k=0}^{L_1-1} C_k^{L_1-1} |\rho|^{2(L_1-k-1)} (1-|\rho|^2)^k \times \sum_{l=1}^{N_I} \eta_l \left(1 - \bar{\gamma}_{3l}^{L_1-k} z^{L_1-k}\right) \left(1 - \frac{z^{L_2}}{L_2!\mu^{L_2}} + o\left(z^{L_2+1}\right)\right). \tag{23}$$

Taking the first-order terms of the McLaurin expansion of $F_{\gamma_{\text{DUB}}}(z)$, it can be approximated as

$$F_{\gamma_{\text{DUB}}}(z) \approx (1-|\rho|^2)^{L_1-1} \sum_{l=1}^{N_I} \eta_l \bar{\gamma}_{3l} z + o(z^2). \tag{24}$$

By substituting $z = \gamma/\bar{\gamma}_1$ into (21) and (24), the proof for *Theorem* 1 is completed. ■

B. Symbol Error Rate at High SNR

Considering only the interference, i.e., $\rho = 1$, we derive the asymptotic SER using [13] and (17) as (detailed derivation is omitted here due to space limitations)

$$P_E^{\infty} \approx \begin{cases} \frac{a\Gamma\left(L_1+\frac{1}{2}\right)}{2\sqrt{\pi} b^{L_1} \bar{\gamma}_1^{L_1}} \sum_{l=1}^{N_I} \eta_l \bar{\gamma}_{3l}^{L_1}, & L_1 < L_2 \\ \frac{a\Gamma\left(L_2+\frac{1}{2}\right)}{2\sqrt{\pi} b^{L_2} \bar{\gamma}_1^{L_2}} \sum_{l=1}^{N_I} \frac{\eta_l}{L_2!\mu^{L_2}}, & L_1 > L_2 \\ \frac{a\Gamma\left(L+\frac{1}{2}\right)}{2\sqrt{\pi} b^{L} \bar{\gamma}_1^{L}} \sum_{l=1}^{N_I} \eta_l \left(\bar{\gamma}_{3l}^{L} + \frac{1}{L!\mu^L}\right), & L_1 = L_2 = L \end{cases}. \tag{25}$$

Our result in (25) indicates that under the effect of interference, the diversity gain is $\min(L_1, L_2)$.

Considering both interference and feedback delay, the SER behavior in the high SNR regime is expressed as

$$P_E \approx \frac{ab^{-1}}{4\bar{\gamma}_1} (1-|\rho|^2)^{L_1-1} \sum_{l=1}^{N_I} \eta_l \bar{\gamma}_{3l}. \tag{26}$$

We observe from (26) that under the combined effect of interference and feedback delay, the diversity gain is unity. This result is independent of the number of antennas at the source and the destination.

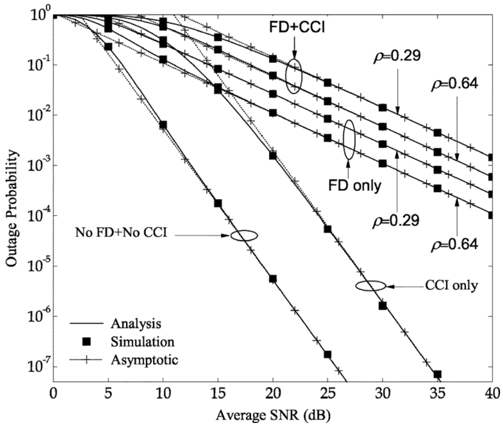

Fig. 1. OP versus the average SNR for different feedback delays.

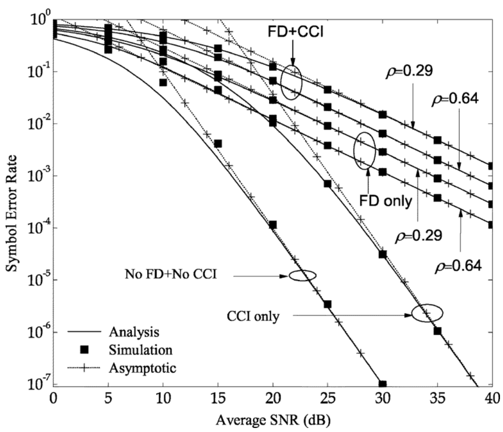

Fig. 2. SER versus the average SNR for various feedback delays.

V. Numerical Results

We present the numerical results to verify our analysis and investigate the impact of feedback delay and interference on the system performance. The system parameters are set as $\Omega_1 = \Omega_2 = 1$, $L_1 = 3$, $L_2 = 4$, $P_l = 5$ dB. Here, we consider three interferers, i.e., $N_I = 3$, with $\{\Omega_{3l}\}_{l=1}^{3} = \{0.4,\ 0.5,\ 0.8\}$ and the feedback delay parameters are set as $\rho = 1$, 0.642, 0.290. The SNR threshold is selected as $\gamma_{\rm th} = 5$ dB and 8-PSK modulation is used. In the following, we assume that $\sigma_1^2 = \sigma_2^2 = \sigma_n^2$, $P_r = P_0$, and define the average SNR as $\bar{\gamma} = P_0/\sigma_n^2$.

Figs. 1 and 2 show the OP and the SER, respectively, versus the average SNR for various normalized correlation coefficients of feedback delay ρ. In particular, we present four different scenarios, i.e., feedback delay and co-channel interference ("FD+CCI"), feedback delay only ("FD only"), co-channel interference only ("CCI only"), and no feedback delay and no co-channel interference ("No FD+No CCI"). As can clearly be seen from Figs. 1 and 2, the simulation is in close agreement with analysis and the asymptotic result is very tight with the exact curve in the high SNR regime, which verifies the correctness of our analysis. It can be seen that high feedback delay, i.e., small ρ, significantly degrades the network performance. As expected, the best performance can be achieved for the ideal case, i.e., without feedback delay and without interference. The considered network provides a diversity order of three without feedback delay whereas only unity diversity gain can be achieved for the cases of feedback delay.

VI. Conclusions

A dual-hop beamforming AF relay network is investigated under the combined effect of interference and feedback delay. Specifically, we derived closed-form expressions for the OP and SER. In addition, asymptotic expressions, which tightly converge to the analytical results in the high SNR regime, are presented for network diversity evaluation. For interference with no feedback delay, the diversity gain is the minimum of the number of antennas at the source and destination. However, under the combined effect of interference and feedback delay, the achievable diversity gain is one regardless of the number of antennas of the networks.

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